Exercise Set 1

SE Modality in Logic and Language

Deadline for submission: 4 April 2018, 11:30am[1]

Please answer <u>all</u> of the following questions. Total of 33 Points:

1. Use Definition 1.1 (*Handout Propositional Logic*) to decide whether the following are well-formed formulae. Explain your answers. 2 Points

   (a) $((p \supset q) \supset (\neg q \supset \neg p))$
   (b) $p \wedge \neg p \supset p$
   (c) $A \supset (B \supset A)$
   (d) $\bot \supset q$

2. Fill in the quotation marks where necessary to make the following sentences true: 2 Points

   (a) Wien is what Wien refers to.
   (b) Consist of five words consists of several words.
   (c) There are seven words in this sentence.
   (d) Wien refers to Wien is a sentence about what Wien means.

3. Check the truth of each of the following, using tableaux. If the inference is invalid, read off a counter-model from the tree, and check directly that it makes the premises true and the conclusion false: 4 Points

   (a) $p \supset q, r \supset q \vdash_{\mathsf{C}} (p \vee r) \supset q$
   (b) $\vdash_{\mathsf{C}} ((p \supset q) \supset q) \supset q$

4. Show that the truth value of $\neg\Box A$ at a world is the same as that of $\Diamond\neg A$. (Hint: Use the clauses for $\Box$, $\Diamond$, and $\neg$ of the definition of a valuation for a model of propositional modal logic on Handout *(2) Propositional Modal Logic I*.) 4 Points

5. Call a world **blind** if it sees no worlds. If a world w is blind, what type of formula is vacuously true? Which is vacuously false? 3 Points

6. Consider again the definition of validity in system K (Definition 3.4 on *Handout (3)*):

   We say that a world w of model $\mathcal{M}(= \langle \mathcal{W}, \mathcal{R}, \mathcal{J} \rangle)$ **models** formula A just in case the given formula is true at that world on that model, i.e. $\nu_{\mathcal{M},w}(A) = 1$.

   Let $\mathcal{M}$ be a model $\langle \mathcal{W}, \mathcal{R}, \mathcal{J} \rangle$. We say that a formulae A is **true in** $\mathcal{M}$ iff for every world $w \in \mathcal{W}$, $\nu_{\mathcal{M},w}(A) = 1$.

   Using K (for Kripke) to refer to our basic modal logic, we say that an inference is **valid in system K** iff every world of every model that models the premises also models the conclusion; i.e.

[1] You can submit your answers in person before class, or you can email me an electronic scan of your answers by 11:30am.

$$\Sigma \vDash_{\mathsf{K}} A \text{ iff for all worlds } w \in W \text{ of all models } \langle \mathcal{W}, \mathcal{R}, \mathcal{I} \rangle\text{:}$$
$$\text{if } \nu_{\mathcal{M},w}(B) = 1 \text{ for all the premises } B \in \Sigma, \text{ then } \nu_{\mathcal{M},w}(A) = 1$$

Exercise: Rewrite the definition of validity in system K ('an inference is valid in system K iff ...') by using the notion of *truth in model* $\mathcal{M}$ (as defined) instead of the notion of a world *modeling* a formula on the right-hand side of the biconditional. (Rewrite it in such a way that it is equivalent to the definition as stated above.) 2 Points

7. The formula $\Box p \supset \Diamond p$ is not valid in system K (i.e. $\nvDash_{\mathsf{K}} \Box p \supset \Diamond p$).

   (a) Find a model $\mathcal{M}(= \langle \mathcal{W}, \mathcal{R}, \mathcal{I} \rangle)$ that invalidates $\Box p \supset \Diamond p$ (i.e. a counter-model to $\vDash_{\mathsf{K}} \Box p \supset \Diamond p$). Draw a diagram of the model (cf. Priest 2008, §§2.3 and 2.4.8). (Hint: Check §4.1(iv) of *Handout (2)* for a relevantly similar example.) 3 Points

   (b) Does this fact about K make it a suitable logic for necessity? Why or why not? (Answer in no more than 200 words.) 3 Points

8. Test the following, using tableaux. Where the tableau does not close, use it to define a counter-model, and draw this, as in Priest (2008, §2.4.8). 10 Points

   (a) $\vdash_{\mathsf{K}} (\Box p \wedge \Box q) \supset \Box(p \wedge q)$

   (b) $\vdash_{\mathsf{K}} \Diamond(p \wedge q) \supset (\Diamond p \wedge \Diamond q)$

   (c) $\Box p, \Box\neg q \vdash_{\mathsf{K}} \Box(p \supset q)$

   (d) $\Diamond p, \Diamond q \vdash_{\mathsf{K}} \Diamond(p \wedge q)$

Exercise Set 2

SE Modality in Logic and Language

Deadline for submission: Thursday, 2 May 2019, 11:30am

Please answer all of the following questions.

1. (a) Describe a **K**-model in which '$\Box p \supset \Diamond p$' is false.
   (b) Describe a **T**-model in which '$\Box p \supset \Box\Box p$' is false.
   (c) Describe a **D**-model that is not a **T**-model.
   (d) Describe a **S4**-model in which '$\Diamond p \supset \Box\Diamond p$' is false.
   (e) Describe a **S4**-model in which '$\Diamond\Box p \supset p$' is false.
   (f) Describe a **B**-model in which '$\Diamond p \supset \Box\Diamond p$' is false.
   (g) Describe a **S5**-model in which '$\Diamond p \supset \Box p$' is false.
   (h) Describe a B-model in which '$\Box p \supset \Box\Box p$' is false.

2. Use tableaux to show that the following argument is derivable in T:
   $\Box(p \supset q), \Box(q \supset r), \Box(r \supset s), \neg\Diamond s \therefore \neg\Diamond p$

3. Give proofs using tableaux for the following wffs in S5:
   (a) $\Diamond\Diamond p \supset \Diamond p$
   (b) $\Diamond(p \vee q) \equiv (\Diamond p \vee \Diamond q)$

4. For each of the following formulas, determine whether it is a logical truth of T, S4, and/or S5. Give countermodels when a formula is not a logical truth of a system, tableaux proofs when it is. (Check each formula against all three systems. If the same tableau proof can be given in two systems, you only need to write it down once and state that it also holds in the other system.)
   (a) $\Box(p \supset \Box\Diamond p)$
   (b) $\Diamond(p \vee q) \supset \Diamond p$
   (c) $\Diamond\Box p \supset \Box p$
   (d) $\Diamond\Box\Diamond p \supset \Diamond p$

5. Show by semantic reasoning that the following wff is a logical truth in K:
   $\vDash_K \Box(\neg(p \supset q) \supset (p \wedge \neg q))$

6. (a) Explain in your own words what makes a system of modal propositional logic *normal*.
   (b) When given two systems of normal modal propositional logic, how can you determine whether one is an extension of the other?

Exercise Set 3

SE Modality in Logic and Language

Deadline for submission: Thursday, 23 May 2019, 3-4pm in my office, C0220

Please answer all of the following questions. Total of 34 Points:

1. Explain the difference between the extension and intension of an expression. Explain what extensions and intensions are. 3 Points

2. State and explain in your own words: 5 Points

   (a) What are the semantic types of the extension and intension of a sentence?

   (b) Explain what an intensional operator is? Give an example of an intensional operator in German (or your native language) and state its semantic entry.

3. Consider the lexical entry for 'and' in von Fintel & Heim (2011, p. 6): 3 Points

   (14a) $[\![\textbf{and}]\!]^{w,g} = \lambda u \in \mathrm{D_t}.\ \lambda v \in \mathrm{D_t}.\ u = v = 1$

   Is this entry extensionally equivalent to conjunction in propositional logic? That is, given the extensions of two sentences as inputs, does the function yield the same extension for the conjunctive sentence as the conjunction junctor would given two truth values of inputting formulas? *Show* why or why not. (Reminder: $\nu_{\mathcal{J}}(A \wedge B) = 1$ iff $\nu_{\mathcal{J}}(A) = 1$ and $\nu_{\mathcal{J}}(B) = 1$)

4. State the lexical entries for the following expressions, following the entries in (13a-c) in von Fintel & Heim (2011, p. 6): 4 Points

   (a) 'popular'
   (b) 'mayor'
   (c) 'go to'
   (d) 'like' (the transitive verb, as used in 'Jill likes Finn')

5. Give lexical entries for the following attitude verbs, following the entry for 'believe' in von Fintel & Heim (2011, (33) on p. 20): 3 Points

   (a) 'assume'
   (b) 'hope'
   (c) 'doubt'

6. Possible-worlds analyses of attitude verbs have notorious problems with pairs like the following: 4 Points

   (1) Lisa believes that a triangle whose three sides are of equal length is equilateral.
   (2) Lisa doesn't believe that a triangle whose three sides are of equal length is equiangular.

   (3) Cem knows that 2+2=4.
   (4) Cem doesn't know that $\pi = 3.14159\ldots$

   Can you explain what the problem here is? [Hint: Think of similarities and differences in intension of the (simple and complex) expressions in the prejacents.]

7. Come up with examples of epistemic, deontic, and circumstantial uses of the necessity verb *have to*. Describe the set of worlds that constitutes the understood restrictor in each of your examples. (This is exercise 3.1 in von Fintel & Heim, 2011, p. 37.) 5 Points

8. Let us call an accessibility relation TRIVIAL if it makes every world accessible from every world. R is TRIVIAL iff $\forall w \forall w' : w' \in R(w)$. What would the conversational background f have to be like for the accessibility relation R_f to be trivial in this sense? (This is exercise 3.6 in von Fintel & Heim, 2011, p. 42.) 3 Points

9. Kratzer argues that because of sentences like (104) (von Fintel & Heim, 2011, 59), modal auxiliaries like 'must' are sensitive to two context-dependent parameters ('doubly relative modality'). 4 Points

   (104) John must pay a fine.

   Provide another example sentence (including a context–cf. von Fintel & Heim (2011, 59-60), state the modal flavours involved in modal base and ordering source as well as the domain of quantification that results from the restrictions by modal base and ordering source.

References

von Fintel, K. & Heim, I. (2011). Intensional semantics. Unpublished Lecture Notes.

Exercise Set 1 — Solutions

SE Modality in Logic and Language

Dr. Dirk Kindermann

May 9, 2019

1. Use Definition 1.1 (*Handout Propositional Logic*) to decide whether the following are well-formed formulae. Explain your answers.

   (a) '$((p \supset q) \supset (\neg q \supset \neg p))$' is a wff.

   (b) '$p \wedge \neg p \supset p$' is, strictly speaking, not a wff; in contrast, '$(p \wedge \neg p) \supset p$' is a wff, and so are '$p \wedge (\neg p \supset p)$' and '$p \wedge \neg (p \supset p)$' (we assume the convention of dropping outermost brackets)

   (c) '$A \supset (B \supset A)$' is not a wff: 'A' and 'B' are *meta*variables, ranging over wffs; they are not included in the (object) language according to Definition 1.1. For (c) to become a wff, the metavariables have to be replaced either by lowercase letters 'p', 'q', ... or by any other wff in parentheses or fronted by a negation sign.

   (d) '$\bot \supset q$' is not a wff in the language defined in Definition 1.1. (In other languages for propositional logic, '$\bot$' stands for the falsum, or absurdity (i.e., by definition, it assumes the truth value *false*).

2. Fill in the quotes where necessary to make the following sentences true:

   (a) Wien is what 'Wien' refers to.

   (b) 'Consist of five words' consists of several words.

   (c) There are seven words in this sentence.

   (d) "Wien' refers to Wien' is a sentence about what 'Wien' means.

3. Check the truth of each of the following, using tableaux. If the inference is invalid, read off a counter-model from the tree, and check directly that it makes the premises true and the conclusion false:

   (a) $p \supset q, r \supset q \vdash_{\mathsf{c}} (p \vee r) \supset q$

$$
\begin{array}{c}
p \supset q \checkmark \\
r \supset q \checkmark \\
\neg((p \vee r) \supset q) \checkmark \\
| \\
p \vee r \checkmark \\
\neg q \\
\begin{array}{cc}
\begin{array}{c}
\neg p \\
\begin{array}{cc}
\begin{array}{c} p \\ \times \end{array} &
\begin{array}{c} r \\ \begin{array}{cc} \neg r & q \\ \times & \times \end{array} \end{array}
\end{array}
\end{array}
&
\begin{array}{c} q \\ \times \end{array}
\end{array}
\end{array}
$$

   We have shown that $p \supset q, r \supset q \vdash_{\mathsf{c}} (p \vee r) \supset q$.

(b) $\vdash_{\mathsf{C}} ((p \supset q) \supset q) \supset q$

$$
\begin{array}{c}
\neg(((p \supset q) \supset q) \supset q) \ \checkmark \\
| \\
(p \supset q) \supset q \ \checkmark \\
\neg q \\
\begin{array}{cc}
\neg(p \supset q)\checkmark & q \\
| & \times \\
p & \\
\neg q & \\
\uparrow &
\end{array}
\end{array}
$$

There is an open branch on the tree. The inference is invalid: $\nvdash_{\mathsf{C}} ((p \supset q) \supset q) \supset q$
Counterexample: $\nu(p) = 1$, $\nu(q) = 0$.
Check directly that it makes the premises true and the conclusion false:

$((p$	$\supset$	$q)$	$\supset$	$q)$	$\supset$	q
1		0		0		0
	0					
			1			
					$\underline{0}$	

4. Show that the truth value of $\neg\Box A$ at a world is the same as that of $\Diamond\neg A$. (Hint: Use the clauses for $\Box, \Diamond$, and $\neg$ of the definition of a valuation for a model of propositional modal logic on Handout *(2) Propositional Modal Logic.*)

   Answer: We will need the following clauses of the definition of a valuation:

   Definition 3.3 Where $\mathcal{M}(= \langle \mathcal{W}, \mathcal{R}, \mathcal{J} \rangle)$ is any model for modal propositional logic, the **valuation** for $\mathcal{M}$, $\nu_{\mathcal{M}}$, is defined as the two-place function that assigns either 0 or 1 to each wff relative to each member of $\mathcal{W}$, subject to the following constraints, where α is any propositional letter, A and B are any wffs, and w is any member of $\mathcal{W}$:

$$
\begin{array}{rcl}
\nu_{\mathcal{M},w}(\alpha) & = & \mathcal{J}_{\mathcal{M},w}(\alpha) \\
\nu_{\mathcal{M},w}(\neg A) = 1 & \text{iff} & \nu_{\mathcal{M},w}(A) = 0 \\
& \vdots & \\
\nu_{\mathcal{M},w}(\Box A) = 1 & \text{iff} & \nu_{\mathcal{M},x}(A) = 1 \text{ at all worlds } x \text{ such that } w\mathcal{R}x \\
\nu_{\mathcal{M},w}(\Diamond A) = 1 & \text{iff} & \nu_{\mathcal{M},x}(A) = 1 \text{ at some world } x \text{ such that } w\mathcal{R}x
\end{array}
$$

   Now, take an *arbitrary* world w:

$$
\begin{array}{rlll}
\nu_{\mathcal{M},w}(\neg\Box A) = 1 & \text{iff} & \nu_{\mathcal{M},w}(\Box A) = 0 & \text{(by clause for } \neg) \\
& \text{iff} & \nu_{\mathcal{M},x}(A) = 0 \text{ for some world } x \text{ s.t. } w\mathcal{R}x & \text{(by clause for } \Box) \\
& \text{iff} & \nu_{\mathcal{M},x}(\neg A) = 1 \text{ for some world } x \text{ s.t. } w\mathcal{R}x & \text{(by clause for } \neg) \\
& \text{iff} & \nu_{\mathcal{M},w}(\Diamond\neg A) = 1. & \text{(by clause for } \Diamond)
\end{array}
$$

   Since w is an *arbitrary* world, the result holds for any world. (Cf. Priest (2008 §2.3.9) for the proof that $\nu_{\mathcal{M},w}(\Box\neg A) = 1$ iff $\nu_{\mathcal{M},w}(\neg\Diamond A) = 1$.)

5. Call a world **blind** if it sees no worlds. If a world w is blind, what type of formula is vacuously true? Which is vacuously false?

   Answer: If w sees no world, then $\Box A$ is vacuously true at w. Given the definition of valuation, $\Box A$ is true at w (relative to model $\mathcal{M}$) iff it is true at all worlds x such that $w\mathcal{R}x$; but since there is no such world, it is vacuously true that the formula is true at *all* such worlds (cf. the parallel to the semantics of the universal quantifier if we allow the domain of quantification to be empty).

$\Diamond A$ is vacuously false at w. $\Diamond A$ is true at a world w iff it is true at *some* accessible world – i.e. iff $\nu_{\mathcal{M},x}(A) = 1$ at some world x such that $w\mathcal{R}x$. Since there is no such world, the formula is false no matter what we put in the place of 'A'.

6. Rewrite the definition of validity in system K (Definition 3.4 on *Handout (2)*) by using the notion of *truth in model* $\mathcal{M}$ (as defined) instead of the notion of a world *modeling* a formula on the right-hand side of the biconditional. (Rewrite it in such a way that it is equivalent to the definition as stated above.)

   Answer: We write 'Using K (for Kripke) to refer to our basic modal logic, we say that an inference is valid in system K iff for every model $\mathcal{M}$, whenever each premise is true in $\mathcal{M}$, the conclusion is true in $\mathcal{M}$.' The rest of Definition 3.4. remains as it is.

7. The formula $\Box p \supset \Diamond p$ is not valid in system K (i.e. $\nvDash_{\mathsf{K}} \Box p \supset \Diamond p$).

   (a) Find a model $\mathcal{M}(= \langle \mathcal{W}, \mathcal{R}, \mathcal{J}\rangle)$ that invalidates $\Box p \supset \Diamond p$ (i.e. a counter-model to $\vDash_{\mathsf{K}} \Box p \supset \Diamond p$). Draw a diagram of the model (cf. Priest 2008, §§2.3 and 2.4.8). (Hint: Check §4.1(iv) of *Handout III-1* for a relevantly similar example.)

   Answer: Counter-model: $\langle \mathcal{W}, \mathcal{R}, \mathcal{J}\rangle$ with
   $\mathcal{W} = \{w_1\}$
   $\mathcal{R} = \emptyset$ ($\mathcal{R}$ doesn't relate w_1 to any world)
   $\mathcal{J}$ can be any function from propositional letters and worlds to truth values.

$$\begin{array}{c} w_1 \\ \neg(\Box p \supset \Diamond p) \\ \Box p \\ \neg\Diamond p \end{array}$$

   [$\nu_{\mathcal{M},w_1}(\Box p) = 1$ since it is trivially the case that at all worlds x accessible from w_1, $\nu_{\mathcal{M},x}(p) = 1$ (w_1 accesses *no* world). But $\nu_{\mathcal{M},w_1}(\neg\Diamond p) = 0$ and hence $\nu_{\mathcal{M},w_1}(\Diamond p) = 0$ because there is no world x accessible from w_1 s.t. $\nu_{\mathcal{M},x}(p) = 1$. Thus, $\nu_{\mathcal{M},w_1}(\neg(\Box p \supset \Diamond p)) = 1$ and $\nu_{\mathcal{M},w_1}(\Box p \supset \Diamond p) = 0$, as its antecedent is true at w_1 and its consequent false.]

   (b) Does this fact about K make it a suitable logic for necessity? Why or why not? (Answer in no more than 200 words.)

   Answer: On the familiar notion of necessity, something has to be possible (relative to our world) for it to be necessarily the case. This makes K an unsuitable logic for necessity. It is too weak – a stronger logic on which $\Box p \supset \Diamond p$ is valid is needed.) For instance, if it is necessary that every mammal has biological parents, it should also be possible for all mammals to biological parents. Something seems to be going wrong if someone says 'Mammals must have biological parents, although mice cannot have biological parents.'

8. Test the following, using tableaux. Where the tableau does not close, use it to define a counter-model, and draw this, as in Priest (2008, §2.4.8).

   (a) $\vdash_{\mathsf{K}} (\Box p \wedge \Box q) \supset \Box(p \wedge q)$

$$\begin{array}{c}
\neg((\Box p \wedge \Box q) \supset \Box(p \wedge q)) \quad ,0 \\
\downarrow \\
\Box p \wedge \Box q \quad ,0 \quad \checkmark \\
\neg\Box(p \wedge q) \quad ,0 \quad \checkmark \\
\downarrow \\
\Diamond\neg(p \wedge q) \quad ,0 \quad \checkmark \\
\downarrow \\
0r1 \\
\neg(p \wedge q) \quad ,1 \quad \checkmark \\
\downarrow \\
\Box p \quad ,0 \\
\Box q \quad ,0 \\
\downarrow \\
p \quad ,1 \\
\downarrow \\
q \quad ,1 \\
\swarrow \qquad \searrow \\
\begin{array}{cc}
\neg p \quad ,1 & \qquad \neg q \quad ,1 \\
\times & \qquad \times
\end{array}
\end{array}$$

So $\vdash_{\mathsf{K}} (\Box p \wedge \Box q) \supset \Box(p \wedge q)$

(b) $\vdash_{\mathsf{K}} \Diamond(p \wedge q) \supset (\Diamond p \wedge \Diamond q)$

$$\begin{array}{c}
\neg(\Diamond(p \wedge q) \supset (\Diamond p \wedge \Diamond q)) \quad ,0 \quad \checkmark \\
\Diamond(p \wedge q) \quad ,0 \quad \checkmark \\
\neg(\Diamond p \wedge \Diamond q) \quad ,0 \quad \checkmark \\
\downarrow \\
0r1 \\
p \wedge q \quad ,1 \quad \checkmark \\
\downarrow \\
p \quad ,1 \\
q \quad ,1 \\
\swarrow \qquad \searrow \\
\begin{array}{cc}
\neg\Diamond p \quad ,0 \quad \checkmark & \qquad \neg\Diamond q \quad ,0 \quad \checkmark \\
\downarrow & \qquad \downarrow \\
\Box\neg p \quad ,0 & \qquad \Box\neg q \quad ,0 \\
\downarrow & \qquad \downarrow \\
\neg p \quad ,1 & \qquad \neg q \quad ,1 \\
\times & \qquad \times
\end{array}
\end{array}$$

So $\vdash_{\mathsf{K}} \Diamond(p \wedge q) \supset (\Diamond p \wedge \Diamond q)$

(c) $\Box p, \Box\neg q \vdash_{\mathsf{K}} \Box(p \supset q)$

$$\begin{array}{c}
\Box p \quad ,0 \\
\Box\neg q \quad ,0 \\
\neg\Box(p \supset q) \quad ,0 \quad \surd \\
\downarrow \\
\Diamond\neg(p \supset q) \quad ,0 \quad \surd \\
\downarrow \\
0r1 \\
\neg(p \supset q) \quad ,1 \quad \surd \\
\downarrow \\
p \quad ,1 \\
\neg q \quad ,1 \\
\downarrow \\
p \quad ,1 \\
\downarrow \\
\neg q \quad ,1 \\
\\
\uparrow
\end{array}$$

So $\Box p, \Box\neg q \nvdash_{\mathsf{K}} \Box(p \supset q)$

Counter-model: $\langle \mathcal{W}, R, J \rangle$ s.t.
$\mathcal{W} = \{w_0, w_1\}$
$\mathcal{R} = \{\langle w_0, w_1 \rangle\}$
$\mathcal{J}(p, w_1) = 1, \mathcal{J}(q, w_1) = 0$

$$\begin{array}{ccc}
w_0 & \longrightarrow & w_1 \\
 & & p \\
 & & \neg q
\end{array}$$

(d) $\Diamond p, \Diamond q \vdash_{\mathsf{K}} \Diamond(p \wedge q)$

$$\begin{array}{ccc}
 & \Diamond p \quad ,0 \quad \surd & \\
 & \Diamond q \quad ,0 \quad \surd & \\
 & \neg\Diamond(p \wedge q) \quad ,0 \quad \surd & \\
 & \downarrow & \\
 & \Box\neg(p \wedge q) \quad ,0 \quad /1/2 & \\
 & \downarrow & \\
 & 0r1 & \\
 & p \quad ,1 & \\
 & \downarrow & \\
 & \neg(p \wedge q) \quad ,1 \quad \surd & \\
\swarrow & & \searrow \\
\neg p \quad ,1 & & \neg q \quad ,1 \\
\times & & \downarrow \\
 & & 0r2
\end{array}$$

$$
\begin{array}{ccc}
 & q \quad ,2 & \\
 & \downarrow & \\
 & \neg(p \wedge q) \quad ,2 \quad \surd & \\
\swarrow & & \searrow \\
\neg p \quad ,2 & & \neg q \quad ,2 \\
\uparrow & & \times
\end{array}
$$

So $\Diamond p, \Diamond q \nvdash_{\mathsf{K}} \Diamond(p \wedge q)$

Counter-model: $\langle \mathcal{W}, R, J \rangle$ s.t.
$\mathcal{W} = \{w_0, w_1, w_2\}$
$\mathcal{R} = \{\langle w_0, w_1 \rangle, \langle w_0, w_2 \rangle\}$
$\mathcal{J}(p, w_1) = 1, \mathcal{J}(q, w_1) = 0, \mathcal{J}(p, w_2) = 0, \mathcal{J}(q, w_2) = 1$

$$
\begin{tikzcd}
 & w_1 \quad p, \neg q \\
w_0 \arrow[ur] \arrow[dr] & \\
 & w_2 \quad \neg p, q
\end{tikzcd}
$$

Exercise Set 2 — Solutions

SE Modality in Logic and Language

Dr. Dirk Kindermann

June 18, 2019

1. (a) Describe a **K**-model in which '$\Box p \supset \Diamond p$' is false. Each of a)–h): 1 Point

   $\mathcal{M} = \langle \mathcal{W}, \mathcal{R}, \mathcal{J} \rangle$, where
   $\mathcal{W} = \{w_1\}$,
   $\mathcal{R} = \emptyset$

   (b) Describe a **T**-model in which '$\Box p \supset \Box\Box p$' is false.

   $\mathcal{M} = \langle \mathcal{W}, \mathcal{R}, \mathcal{J} \rangle$, where
   $\mathcal{W} = \{w_1, w_2, w_3\}$,
   $\mathcal{R} = \{\langle w_1, w_1\rangle, \langle w_2, w_2\rangle, \langle w_3, w_3\rangle, \langle w_1, w_2\rangle, \langle w_2, w_3\rangle\}$,
   $\mathcal{J}(p, w_1) = 1$, $\mathcal{J}(p, w_2) = 1$, $\mathcal{J}(p, w_3) = 0$

   (c) Describe a **D**-model that is not a **T**-model.

   Any model whose accessibility relation $\mathcal{R}$ is serial but not reflexive is a **D**-model but not a **T**-model: $\ulcorner \Box A \supset A \urcorner$ is true on every **T**-model but not on the described **D**-model.

   Example: '$\Box p \supset p$' is false on the **D**-model $\mathcal{M} = \langle \mathcal{W}, \mathcal{R}, \mathcal{J} \rangle$, where
   $\mathcal{W} = \{w_1, w_2\}$
   $\mathcal{R} = \{\langle w_1, w_2\rangle, \langle w_2, w_1\rangle\}$,
   $\mathcal{J}(p, w_1) = 0$, $\mathcal{J}(p, w_2) = 1$

   (d) Describe a **S4**-model in which '$\Diamond p \supset \Box\Diamond p$' is false.

   $\mathcal{M} = \langle \mathcal{W}, \mathcal{R}, \mathcal{J} \rangle$, where
   $\mathcal{W} = \{w_1, w_2\}$,
   $\mathcal{R} = \{\langle w_1, w_1\rangle, \langle w_1, w_2\rangle, \langle w_2, w_2\rangle\}$,
   $\mathcal{J}(p, w_1) = 1$, $\mathcal{J}(p, w_2) = 0$

   (e) Describe a **S4**-model in which '$\Diamond\Box p \supset p$' is false.

   $\mathcal{M} = \langle \mathcal{W}, \mathcal{R}, \mathcal{J} \rangle$, where
   $\mathcal{W} = \{w_1, w_2\}$,
   $\mathcal{R} = \{\langle w_1, w_1\rangle, \langle w_1, w_2\rangle, \langle w_2, w_2\rangle\}$,
   $\mathcal{J}(p, w_1) = 0$, $\mathcal{J}(p, w_2) = 1$

   (f) Describe a **B**-model in which '$\Diamond p \supset \Box\Diamond p$' is false.

   $\mathcal{M} = \langle \mathcal{W}, \mathcal{R}, \mathcal{J} \rangle$, where

$\mathcal{W} = \{w_1, w_2, w_3\}$,
$\mathcal{R} = \{\langle w_1, w_1\rangle, \langle w_2, w_2\rangle, \langle w_3, w_3\rangle, \langle w_1, w_2\rangle, \langle w_2, w_1\rangle, \langle w_1, w_3\rangle, \langle w_3, w_1\rangle\}$,
$\mathcal{J}(p, w_1) = 0$, $\mathcal{J}(p, w_2) = 0$, $\mathcal{J}(p, w_3) = 1$

(g) Describe a **S5**-model in which '$\Diamond p \supset \Box p$' is false.

$\mathcal{M} = \langle \mathcal{W}, \mathcal{R}, \mathcal{J}\rangle$, where
$\mathcal{W} = \{w_1, w_2\}$,
$\mathcal{R} = \{\langle w_1, w_1\rangle, \langle w_2, w_2\rangle, \langle w_1, w_2\rangle, \langle w_2, w_1\rangle\}$,
$\mathcal{J}(p, w_1) = 1$, $\mathcal{J}(p, w_2) = 0$

(h) Describe a **B**-model in which '$\Box p \supset \Box\Box p$' is false.

$\mathcal{M} = \langle \mathcal{W}, \mathcal{R}, \mathcal{J}\rangle$, where
$\mathcal{W} = \{w_1, w_2, w_3\}$,
$\mathcal{R} = \{\langle w_1, w_1\rangle, \langle w_2, w_2\rangle, \langle w_3, w_3\rangle, \langle w_1, w_2\rangle, \langle w_2, w_1\rangle, \langle w_2, w_3\rangle, \langle w_3, w_2\rangle\}$,
$\mathcal{J}(p, w_1) = 1$, $\mathcal{J}(p, w_2) = 1$, $\mathcal{J}(p, w_3) = 0$

2. Use tableaux to show that the following argument is derivable in T:
$\Box(p \supset q), \Box(q \supset r), \Box(r \supset s), \neg\Diamond s \;\therefore\; \neg\Diamond p$ 2,5 Points

$$\Box(p \supset q), 0/1$$
$$\Box(q \supset r), 0/1$$
$$\Box(r \supset s), 0/1$$
$$\neg\Diamond s, 0 \checkmark$$
$$\neg\neg\Diamond p, 0$$
$$\downarrow$$
$$\Box\neg s, 0/1$$
$$\downarrow$$
$$\Diamond p, 0$$
$$\downarrow$$
$$0r1$$
$$p, 1$$
$$\downarrow$$
$$\neg s, 1$$
$$\downarrow$$
$$p \supset q, 1 \checkmark$$
$$\swarrow \quad \searrow$$
$$\neg p, 1 \qquad q, 1$$
$$\times \qquad \downarrow$$

$$
\begin{array}{ccccc}
 & & q \supset r, 1 \checkmark & & \\
 & \swarrow & & \searrow & \\
 & \neg q, 1 & & r, 1 & \\
 & \times & & \downarrow & \\
 & & & r \supset s, 1 \checkmark & \\
 & & \swarrow & & \searrow \\
 & & \neg r, 1 & & s, 1 \\
 & & \times & & \times
\end{array}
$$

So $\Box(p \supset q), \Box(q \supset r), \Box(r \supset s), \neg\Diamond s \vdash_{\mathrm{T}} \neg\Diamond p$

3. Give proofs using tableaux for the following wffs in S5:

   (a) $\Diamond\Diamond p \supset \Diamond p$ 2,5 Points

$$
\begin{array}{c}
\neg(\Diamond\Diamond p \supset \Diamond p), 0 \checkmark \\
\downarrow \\
\Diamond\Diamond p, 0 \checkmark \\
\neg\Diamond p, 0 \\
\downarrow \\
\Box\neg p, 0/2 \\
\downarrow \\
0\mathrm{r}1 \\
\Diamond p, 1 \checkmark \\
\downarrow \\
1\mathrm{r}2 \\
p, 2 \\
\downarrow \\
0\mathrm{r}2 \\
\neg p, 2 \\
\times
\end{array}
$$

(b) $\Diamond(p \vee q) \equiv (\Diamond p \vee \Diamond q)$ 2,5 Points

$$\neg(\Diamond(p \vee q) \equiv (\Diamond p \vee \Diamond q)), 0 \checkmark$$

$$\swarrow \qquad\qquad\qquad\qquad \searrow$$

$$\begin{array}{c|c}
\Diamond(p \vee q), 0\checkmark & \neg\Diamond(p \vee q), 0\checkmark \\
\neg(\Diamond p \vee \Diamond q), 0\checkmark & \Diamond p \vee \Diamond q, 0\checkmark \\
\downarrow & \downarrow \\
0r1 & \Box\neg(p \vee q), 0 \\
p \vee q, 1\checkmark & \swarrow \quad \searrow \\
\downarrow & \Diamond p, 0\checkmark \qquad \Diamond q, 0\checkmark \\
\neg\Diamond p, 0\checkmark & \downarrow \qquad\qquad \downarrow \\
\neg\Diamond q, 0\checkmark & 0r1 \qquad\qquad 0r1 \\
\downarrow & p, 1 \qquad\qquad q, 1 \\
\Box\neg p, 0/1 & \downarrow \qquad\qquad \downarrow \\
\downarrow & \neg(p \vee q), 1\checkmark \qquad \neg(p \vee q), 1\checkmark \\
\Box\neg q, 0/1 & \downarrow \qquad\qquad \downarrow \\
\downarrow & \neg p, 1 \qquad\qquad \neg p, 1 \\
\neg p, 1 & \neg q, 1 \qquad\qquad \neg q, 1 \\
\downarrow & \times \qquad\qquad \times \\
\neg q, 1 & \\
\swarrow \quad \searrow & \\
p, 1 \qquad q, 1 & \\
\times \qquad \times &
\end{array}$$

4. For each of the following formulas, determine whether it is a logical truth of T, S4, and/or S5. Give countermodels when a formula is not a logical truth of a system, tableaux proofs when it is. (Check each formula against all three systems. If the same tableau proof can be given in two systems, you only need to write it down once and state that it also holds in the other system.)

   (a) $\Box(p \supset \Box\Diamond p)$ Each of a)–d): 2,5 Points

   - T and S4: Not a logical truth
     Countermodel for both: $\mathcal{M} = \langle \mathcal{W}, \mathcal{R}, \mathcal{I} \rangle$ and w_1, where
     $\mathcal{W} = \{w_1, w_2, w_3\}$
     $\mathcal{R} = \{\langle w_1, w_1\rangle, \langle w_2, w_2\rangle, \langle w_3, w_3\rangle, \langle w_1, w_2\rangle, \langle w_1, w_3\rangle, \langle w_2, w_3\rangle\}$
     ($\mathcal{R}$ is reflexive and serial and transitive, but asymmetric.)
     $\mathcal{I}(p, w_1) = 1$, $\mathcal{I}(p, w_2) = 1$, $\mathcal{I}(p, w_3) = 0$
   - S5: Logical Truth

$$\neg\Box(p \supset \Box\Diamond p), 0 \checkmark$$
$$\downarrow$$
$$\Diamond\neg(p \supset \Box\Diamond p), 0 \checkmark$$
$$\downarrow$$
$$0r1$$

$$\downarrow$$
$$\neg(p \supset \Box\Diamond p), 1 \checkmark$$
$$\downarrow$$
$$p, 1$$
$$\neg\Box\Diamond p, 1 \checkmark$$
$$\downarrow$$
$$\Diamond\neg\Diamond p, 1 \checkmark$$
$$\downarrow$$
$$1\text{r}2$$
$$\downarrow$$
$$\neg\Diamond p, 2 \checkmark$$
$$\downarrow$$
$$\Box\neg p, 2 \ /1$$
$$\downarrow$$
$$0\text{r}0,\ 1\text{r}1,\ 2\text{r}2,\ 1\text{r}0,\ \underline{2\text{r}1},\ 0\text{r}2,\ 2\text{r}0$$
$$\downarrow$$
$$\neg p, 1$$
$$\times$$

(b) $\Diamond(p \vee q) \supset \Diamond p$

It's not a logical truth in either of T, S4, and S5.
Countermodel: $\mathcal{M} = \langle \mathcal{W}, \mathcal{R}, \mathcal{J} \rangle$, where
$\mathcal{W} = \{w_1, w_2\}$
$\mathcal{R} = \{\langle w_1, w_1 \rangle, \langle w_2, w_2 \rangle, \langle w_1, w_2 \rangle, \langle w_2, w_1 \rangle\}$
$\mathcal{J}(p, w_1) = 0$, $\mathcal{J}(p, w_2) = 0$, $\mathcal{J}(q, w_2) = 1$

OR

$\mathcal{M} = \langle \mathcal{W}, \mathcal{R}, \mathcal{J} \rangle$, where
$\mathcal{W} = \{w_1\}$
$\mathcal{R} = \{\langle w_1, w_1 \rangle\}$
$\mathcal{J}(p, w_1) = 0$, $\mathcal{J}(q, w_1) = 1$

(c) $\Diamond\Box p \supset \Box p$

- T and S4: Not a logical truth
  Countermodel: $\mathcal{M} = \langle \mathcal{W}, \mathcal{R}, \mathcal{J} \rangle$, where
  $\mathcal{W} = \{w_1, w_2\}$
  $\mathcal{R} = \{\langle w_1, w_1 \rangle, \langle w_2, w_2 \rangle, \langle w_1, w_2 \rangle\}$
  $\mathcal{J}(p, w_1) = 0$, $\mathcal{J}(p, w_2) = 1$
  The countermodel is serial, reflexive and transitive (but not symmetric, thus not a S5 model).
- S5: It's a logical truth.

$$\neg(\Diamond\Box p \supset \Box p), 0 \checkmark$$
$$\downarrow$$
$$\Diamond\Box p, 0 \checkmark$$
$$\neg\Box p, 0 \checkmark$$
$$\downarrow$$

$\Diamond\neg p, 0$✓
$\downarrow$
0r1
$\Box p, 1/2$
$\downarrow$
0r2
$\neg p, 2$
$\downarrow$
0r0, 1r1, 2r2, <u>1r2</u>, 2r1
$\downarrow$
$p, 2$
x

(d) $\Diamond\Box\Diamond p \supset \Diamond p$

- T: It's not a logical truth
  Countermodel: $\mathcal{M} = \langle \mathcal{W}, \mathcal{R}, \mathcal{J} \rangle$, where
  $\mathcal{W} = \{w_1, w_2, w_3\}$
  $\mathcal{R} = \{\langle w_1, w_1\rangle, \langle w_2, w_2\rangle, \langle w_3, w_3\rangle, \langle w_1, w_2\rangle, \langle w_2, w_3\rangle\}$
  $\mathcal{J}(p, w_1) = 0$, $\mathcal{J}(p, w_2) = 0$, $\mathcal{J}(p, w_3) = 1$
  (The countermodel is serial and reflexive, but not transitive.)
- S4 and S5: It's a logical truth

$\neg(\Diamond\Box\Diamond p \supset \Diamond p), 0$✓
$\downarrow$
$\Diamond\Box\Diamond p, 0$✓
$\neg\Diamond p, 0$✓
$\downarrow$
$\Box\neg p, 0/2$
$\downarrow$
0r1
$\Box\Diamond p, 1/1$
$\downarrow$
1r1
$\downarrow$
$\Diamond p, 1$✓
$\downarrow$
1r2
$p, 2$
$\downarrow$
0r2
$\neg p, 2$
x

5. Show by semantic reasoning that the following wff is a logical truth in K: 3,5 Points
$\vDash_{\mathsf{K}} \Box(\neg(p \supset q) \supset (p \wedge \neg q))$

Let w_0 be an **arbitrary** world of any K-model $\mathcal{M}$.

The proof shows by reductio that $\Box(\neg(p \supset q) \supset (p \wedge \neg q))$ is true at w_0 in $\mathcal{M}$. Since w_0 and $\mathcal{M}$ are arbitrary, the proof effectively shows that $\Box(\neg(p \supset q) \supset (p \wedge \neg q))$ is true at every world of every K-model, hence that it is a logical truth.

Assumption for reductio: $\nu_{\mathcal{M},w_0}(\neg\Box(\neg(p \supset q) \supset (p \wedge \neg q))) = 1$.

In the following, each step is justified by a clause of Definition 3.3 on Handout 2 of *valuation* ν.

$\nu_{\mathcal{M},w_0}(\neg\Box(\neg(p \supset q) \supset (p \wedge \neg q))) = 1$ iff $\nu_{\mathcal{M},w_0}(\Box(\neg(p \supset q) \supset (p \wedge \neg q))) = 0$
iff $\exists w_1$ s.t. $w_0 R w_1$ and $\nu_{\mathcal{M},w_1}(\neg(p \supset q) \supset (p \wedge \neg q)) = 0$
iff $\nu_{\mathcal{M},w_1}(\neg(p \supset q)) = 1$ and $\nu_{\mathcal{M},w_1}(p \wedge \neg q) = 0$
iff $\nu_{\mathcal{M},w_1}(p \supset q) = 0$ and $\nu_{\mathcal{M},w_1}(p) = 0$ and $\nu_{\mathcal{M},w_1}(\neg q) = 0$
iff $\nu_{\mathcal{M},w_1}(p) = 1$ and $\nu_{\mathcal{M},w_1}(q) = 0$ and $\nu_{\mathcal{M},w_1}(p) = 0$ and $\nu_{\mathcal{M},w_1}(q) = 1$

But it is impossible that p and q each be assigned different truth values at the same world in the same model. Hence, by reductio, it is not the case that $\nu_{\mathcal{M},w_0}(\neg\Box(\neg(p \supset q) \supset (p \wedge \neg q))) = 1$. So it's the case that $\nu_{\mathcal{M},w_0}(\Box(\neg(p \supset q) \supset (p \wedge \neg q))) = 1$. Since w_0 was an arbitrary world of an arbitrary K-model, we have proven that $\vDash_{\mathsf{K}} \Box(\neg(p \supset q) \supset (p \wedge \neg q))$. Q.E.D.

6. (a) Explain in your own words what makes a system of modal propositional logic *normal*. 2 Points

Recall Definitions (2.1), 2.2 (extension) and 2.3 (normal system) on Handout 3:

Definition 2.3:

A system of modal logic is **normal** iff it is an extension of K (i.e., iff it is at least as strong as K).

Definition 2.2:

A system of modal logic, K_n, is an **extension** of a system K_m just in case if $\Sigma \vdash_{\mathsf{K}_m} A$, then $\Sigma \vdash_{\mathsf{K}_n} A$. That is, every inference derivable in K_m is derivable in K_n, and every theorem of K_m is a theorem of K_n.

(b) When given two systems of normal modal propositional logic, how can you determine whether one is an extension of the other? 2 Points

Let's assume that the two systems, call them S_1 and S_2, are each given with definitions of $\vDash$, and of $\vdash$ in terms of tree rules. Now suppose system S_2 is an extension of system S_1. Then we have two options to prove this.

Option 1: We show that every inference derivable in S_1 is derivable in S_2, and every theorem of S_1 is a theorem of S_2. We do this by showing that every derivation of S_1 is also a derivation of S_2. This can be achieved by showing that every tree rule in S_1 is also a tree rule in S_2.

Option 2: If S_1 and S_1 are each sound and complete, we can assume that every valid inference in S_1 is a valid inference in S_2, and every wff valid in S_1 is valid in S_2. We do this by showing that every admissible S_2-model is also an admissible S_1-model (cf. the remarks on model theory on p. 2 of Handout 3).

33 Points in total

Exercise Set 3 — Solutions

SE Modality in Logic and Language

Dr. Dirk Kindermann

August 12, 2019

1. Explain the difference between the extension and intension of an expression. Explain what extensions and intensions are. 3 Points

   The extension of an expression (at a world) can roughly be thought of as that in the world to which the expression refers. For instance, the extension of a proper name is the individual bearing the name; the extension of a predicate is the set of individuals to which the predicate applies; the extension of a sentence is a truth value. For any expression α, we write the extension of α in world w as $[\![\alpha]\!]^{w,g}$.

   The intension of an expression is the function mapping a possible world to the expression's extension at that possible world. Hence, a sentence's intension is a function from possible worlds to truth values. Equivalently, we can think if a sentence's intension as a set of possible worlds– those at which the sentence is true. For any expression α, we write the intension of α as $\lambda w. [\![\alpha]\!]^{w,g}$ (pronounced: 'the function that assigns to any world w the extension of α in that world').

2. State and explain in your own words:

   (a) What are the semantic types of the extension and intension of a sentence? 2 Points

   The semantic type of a sentence's extension is t – truth value).

   The semantic type of a sentence's intension is ⟨s,t⟩ – a function from possible worlds (s) to truth values (t).

   (b) Explain what an intensional operator is. Give an example of an intensional operator in German (or your native language) and state its semantic entry. 3 Points

   An intensional operator is an expression that operates on the intension of the expression with which it combines. In our system, an intensional operator is capable of shifting the world parameter of the semantic value of the expression with which it combines. Two examples in German:

   - $[\![$**Gemäß der Herr-der-Ringe Trilogie**$]\!]^{w,g} = \lambda p \in D_{\langle s,t\rangle}$. for all worlds w' that are as described in the Lord-of-the-Rings trilogy, $p(w') = 1$
   - $[\![$**Lisa glaubt**$]\!]^{w,g} = \lambda p \in \mathrm{D}_{\langle s,t\rangle}$. $\forall w'$ compatible with what Lisa believes in w: $p(w') = 1$

3. Consider the lexical entry for 'and' in von Fintel & Heim (2011, p. 6): 3 Points

   (14a) $[\![\textbf{and}]\!]^{w,g} = \lambda u \in \mathrm{D_t}.\ \lambda v \in \mathrm{D_t}.\ u = v = 1$

   Is this entry extensionally equivalent to conjunction in propositional logic? That is, given the extensions of two sentences as inputs, does the function yield the same extension for the conjunctive sentence as the conjunction junctor would given two truth values of inputting formulas? *Show* why or why not. (Reminder: $\nu_{\mathcal{J}}(A \wedge B) = 1$ iff $\nu_{\mathcal{J}}(A) = 1$ and $\nu_{\mathcal{J}}(B) = 1$)

   Yes, (14a) is extensionally equivalent to conjunction in propositional logic. For two functions to be extensionally equivalent, they must map the same arguments/inputs to the same values/outputs. Here is the truth table for $\wedge$ given the arguments u and v:

u	v	$u \wedge v$
1	1	1
1	0	0
0	1	0
0	0	0

Of the four ways of inputting semantic values (1,0) for u and v, the tables shows that only when $u = v = 1$ does the function yield 1. In all other cases, it yields 0.

The same is true of (14a), which states that u and v be mapped to 1 only if $u = v = 1$, and to 0 otherwise. So the functions that are the extensional meaning of **and** in (14a) and $\wedge$ in classical propositional logic are equivalent.

4. State the lexical entries for the following expressions, following the entries in (13a-c) in von Fintel & Heim (2011, p. 6): 4 Points

   (a) $[\![\textbf{popular}]\!]^{w,g} = \lambda x \in \mathrm{D_e}.\ x$ is popular in w
   (b) $[\![\textbf{mayor}]\!]^{w,g} = \lambda x \in \mathrm{D_e}.\ x$ is (a) mayor in w
   (c) $[\![\textbf{go to}]\!]^{w,g} = \lambda x \in \mathrm{D_e}.\ \lambda y \in \mathrm{D_e}.\ y$ goes to x in w
   (d) $[\![\textbf{like}]\!]^{w,g} = \lambda x \in \mathrm{D_e}.\ \lambda y \in \mathrm{D_e}.\ y$ likes x in w

5. Give lexical entries for the following attitude verbs, following the entry for 'believe' in von Fintel & Heim (2011, (33) on p. 20): 3 Points

   (a) $[\![\textbf{assume}]\!]^{w,g} = \lambda p \in \mathrm{D}_{\langle s,t \rangle}.\ \lambda x \in \mathrm{D_e}.\ \forall w'$ compatible with x's assumptions in w: $p(w') = 1$
   (b) $[\![\textbf{hope}]\!]^{w,g} = \lambda p \in \mathrm{D}_{\langle s,t \rangle}.\ \lambda x \in \mathrm{D_e}.\ \forall w'$ compatible with x's hopes in w: $p(w') = 1$
   (c) $[\![\textbf{doubt}]\!]^{w,g} = \lambda p \in \mathrm{D}_{\langle s,t \rangle}.\ \lambda x \in \mathrm{D_e}.\ \forall w'$ compatible with x's doubts in w: $p(w') = 1$

6. Possible-worlds analyses of attitude verbs have notorious problems with pairs like the following: 4 Points

   (1) Lisa believes that all mammals are renates.
   (2) Lisa doesn't believe that all mammals are cordates.

   (3) Cem knows that 2+2=4.
   (4) Cem doesn't know that π = 3.14159…

   Can you explain what the problem here is? [Hint: Think of similarities and differences in intension of the (simple and complex) expressions in the prejacents.]

   [I'm skipping discussion of pairs (1) and (2), since they're not the main issue here.
   You can look up the problem in chapter 9 of my script *Einführung in die formale Semantik* on www.dirkkindermann.com/teaching.html.]

   In short, the problem with (3) and (4) is that their prejacents have the same intensions. How is that a problem?

   Intuitively, (3) and (4) are not contradictory — it can both be true that Cem knows that 2+2=4 and does not know that π = 3.14159…Now, given the meaning of 'not', it can thus be true that Cem knows that 2+2=4 and false that he knows that π = 3.14159…

   Here is the problem on our analysis: Given the principle of compositionality, two complex expressions differ in meaning only if they either contain different simple expressions or combine in different ways. With our semantic entry for 'know', it's plausible that 'Cem knows that' and 'Cem does not know that' combine via Intensional Functional Application with the intensions of their prejacents, '2+2=4' and 'π = 3.14159...' If (3) is true and unnegated (4) is false, they must differ in meaning; hence they must differ in meaning because their prejacents differ in intensions. But do they? Note that '2+2=4' is a necessary truth–it is true at every possible world. So its intension is the function that maps every possible world to truth. Likewise, 'π = 3.14159...' is a necessary truth; its intension is *also* the function that maps every possible world to truth. Hence both prejacents have the same intension (which is sometimes called 'the necessary proposition', since in possible worlds semantics, there is only one necessary proposition). As a result, in our possible worlds analysis, (3) and unnegated (4) do not differ in meaning, so it cannot be the case that (3) is true and (4) is false. But this runs counter to our intuitions about the meanings and possible truth value distribution of (3) and (4).

7. Come up with examples of epistemic, deontic, and circumstantial uses of the necessity verb *have to*. Describe the set of worlds that constitutes the understood restrictor in each of your examples. (This is exercise 3.1 in von Fintel & Heim, 2011, p. 37.) 5 Points

   - Epistemic reading:

     [Context: Lisa sees Dorothee's car in the driveway]

     Lisa: 'Dorothee has to be at home.'

     Restrictor set: {w: Dorothee's car is in the driveway in w}

   - Deontic reading:

     'According to veganism, you have to refrain from eating animal products.'

     Restrictor set (explicitly given by 'according to veganism': {w: w is just like veganism prescribes}

   - Circumstantial reading:

     'A sunflower has to get sunshine to survive.'

     Restrictor set of the sentence when evaluated at w: {w': the laws of nature of w hold in w'}

8. Let us call an accessibility relation TRIVIAL if it makes every world accessible from every world. R is TRIVIAL iff $\forall w \forall w' : w' \in R(w)$. What would the conversational background f have to be like for the accessibility relation R_f to be trivial in this sense? (This is exercise 3.6 in von Fintel & Heim, 2011, p. 42.) 3 Points

   Intuitively, the conversational background would have to be one of complete ignorance (if it is an epistemic conversational background). If it is trivial, it doesn't restrict the universe (the set of all worlds) at all; so no possibilities are excluded.

   Conversational backgrounds for Kratzer are functions from worlds to sets of propositions; propositions are sets of worlds. Hence, conversational backgrounds are of type ⟨s, ⟨st,t⟩⟩, functions from worlds to (characteristic functions of) sets of propositions.

   A trivial conversational background maps any world w to the empty set (of propositions), $\emptyset$.

   Trivial conversational background: $\lambda w.\ \lambda p.\ p = \emptyset$. (The function that maps world w and proposition p to 1 iff p is the empty set)

9. Kratzer argues that because of sentences like (104) (von Fintel & Heim, 2011, 59), modal auxiliaries like 'must' are sensitive to two context-dependent parameters ('doubly relative modality'). 4 Points

   (104) John must pay a fine.

Provide another example sentence (including a context–cf. von Fintel & Heim (2011, 59-60), state the modal flavours involved in modal base and ordering source as well as the domain of quantification that is restricted by modal base and ordering source.

(⋆) The shop lifter must be sentenced.

The modal base in our example is the proposition that there is a unique person who did the shop lifting. Its modal flavour is circumstantial (or epistemic).

The ordering source is the set of propositions whose truth is demanded by criminal law. Which criminal law is relevant is determined by context of utterance of (⋆): it will have to determine under which jurisdiction the shop lifting falls.

'Must' in (⋆) thus quantifies over the set of worlds which are the best worlds, given criminal law (ordering source), of all the worlds that are compatible with there being a unique person who did the shop lifting (modal base).

34 Points in total

References

von Fintel, K. & Heim, I. (2011). Intensional semantics. Unpublished Lecture Notes.